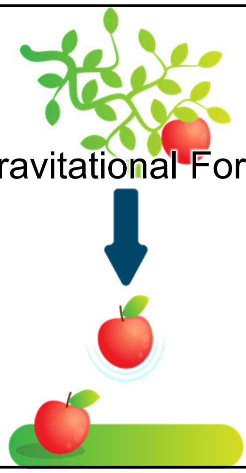
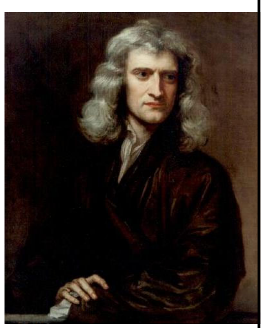
Gravitational Force



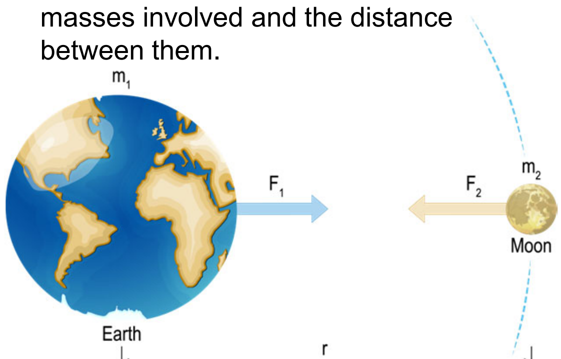
VectorMine (Adobe Stock)

- Sir Isaac Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions.



Godfrey Kneller (public domain)

- The gravitational force is always attractive, and it depends only on the masses involved and the distance between them.



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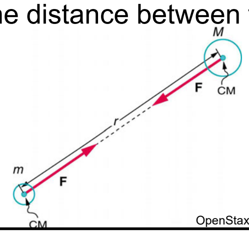
Newton's Law of Universal Gravitation

- Every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.



- The magnitude of the force on each object (one has larger mass than the other) is the same, consistent with Newton's third law.
- The bodies we are dealing with tend to be large.
- To simplify the situation, we assume that the body acts as if its entire mass is concentrated at one specific point called the center of mass (CM).

- For two bodies having masses m_1 and m_2 with a distance r between their centers of mass, the magnitude of the gravitational force between them is

$$\boxed{\left|\vec{F}_g\right| = G\frac{m_1 m_2}{r^2}}$$

G is the universal gravitational constant.

$$\boxed{G = 6.67 \times 10^{-11}\ m^3/kg \cdot s^2}$$

Example 1

- What is the magnitude of the force acting on a 2000 kg spacecraft when it orbits the Earth at a distance of twice the Earth's radius?
  - Radius of Earth = 6380 km
  - Mass of Earth = $5.98\text{x}10^{24}$ kg

$$|\vec{F}_g| = G\frac{m_1 m_2}{r^2}$$

$$|\vec{F}_g| = (6.67\times10^{-11})\frac{(2000)(5.98\times10^{24})}{(2\cdot 6380\times10^3)^2} = 4900\text{ N}$$

Example 2

- Two 70 kg people are sitting on a bench so that their centers of mass are 50 cm apart. What is the magnitude of the gravitational force each exerts on the other?

$$\left|\vec{F}_g\right| = G\frac{m_1 m_2}{r^2}$$

$$\left|\vec{F}_g\right| = (6.67 \times 10^{-11})\frac{(70)(70)}{(0.5)^2} = 1.3 \times 10^{-6}\ \mathrm{N}$$

Gravitational Field Strength

- We can use Newton's Law of Universal Gravitation to calculate the gravitational field strength (also called gravitational acceleration), g.
- The weight of an object of mass M is defined as the gravitational force on the object.

$$F = mg = G\frac{m m_{earth}}{r^2}$$

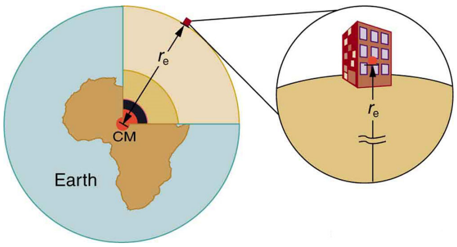
$$g = G\frac{m_{earth}}{r^2}$$

Substituting in the mass and radius of earth gives

$$g = (6.67 \times 10^{-11})\frac{(5.98 \times 10^{24})}{(6.38 \times 10^{6})^2}$$

$$g = 9.80\ m/s^2$$

- The distance between the centers of mass of Earth and an object on its surface is very nearly the same as the radius of Earth, because Earth is so much larger than the object.



Example

- Mt. Everest is 8848 m above sea level. Determine the gravitational field strength at the top of the mountain.
  - Radius of Earth = 6.30×10^6 m
  - Mass of Earth = 5.98×10^{24} kg

$$g = G\frac{m_{earth}}{r^2}$$

$$g = (6.67 \times 10^{-11})\frac{5.98 \times 10^{24}}{(6.38 \times 10^6 + 8848)^2}$$

$$g = 9.77\ m/s^2$$

Inertial Mass & Gravitational Mass

- **Inertial mass** is defined by Newton's Second Law, where a known force is applied to the mass and the acceleration it induces is measured.

$$m = \frac{F}{a}$$

- Gravitational mass Gravitational mass is defined by Newton's Law of Universal Gravitation.

$$F = G\frac{m_1 m_2}{r^2}$$

- It is measured by comparing the force of gravity of an unknown mass to the force of gravity of a known mass. This is typically done with some sort of balance scale.

- No difference has been found between gravitational and inertial mass.
- Many experiments have been performed to check the values and the experiments always agree to within the margin of error for the experiment.
- Einstein used the fact that gravitational and inertial mass were equal to begin his Theory of General Relativity in which he postulated that gravitational mass was the same as inertial mass.